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**Polarized Gluons in the Nucleon \***T. Morii<sup>1,2</sup>, S. Tanaka<sup>1</sup> and T. Yamanishi<sup>2</sup><sup>1</sup>Faculty of Human Development, Kobe University, Nada, Kobe 657, Japan<sup>2</sup>Graduate School of Science and Technology, Kobe University, Nada, Kobe 657, Japan**Abstract**

QCD suggests that gluons in the nucleon play an important role in *so-called* “the proton spin problem”. In this talk, the behavior of the polarized gluon distribution in the nucleon is discussed by using the positivity condition of distribution functions together with the unpolarized and polarized experimental data.

**1. Introduction**

The proton spin problem issued from the measurement of the polarized structure function of proton  $g_1^p(x)$  by the EMC group[1], has caused a great interest in particle and nuclear physics community. The EMC[1] and SMC[2] data on  $g_1^p(x)$  suggest that very little of the proton spin is carried by quarks ( $\Delta\Sigma \approx 0$ ) and furthermore the polarized  $s$ -quark density is rather large ( $\Delta s \approx -0.12$ [2]). The result is apparently incompatible with the prediction of the naive quark and/or parton model. So far a number of theoretical approaches have been proposed to solve the problem. Among these approaches, there is an interesting idea that gluons contribute significantly to the proton spin through the  $U_A(1)$  anomaly of QCD[3]. In this model the polarized quark densities taking part in  $g_1^p$  should be modified as  $\Delta q \rightarrow \Delta q - \frac{\alpha_s}{2\pi} \Delta G$ , where  $\Delta q$  and  $\Delta G$  are polarized quark and gluon densities, respectively, and one can reproduce the EMC/SMC data quite well by taking rather large  $\Delta G (\approx 5 \sim 6)$ [4]. In addition, such a large  $\Delta G$  can solve an apparent discrepancy on the magnitude of  $\Delta s$  between rather large  $\Delta s$  derived from EMC/SMC data and a restrictive bound  $|\Delta s| \leq 0.05_{-0.05}^{+0.02}$ [5] derived from the experiment on charm productions in neutrino deep inelastic scatterings[6]. These observations tell us that it is very important to

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determine the behavior of the polarized gluon distribution  $\delta G(x)$  and its magnitude  $\Delta G (= \int_0^1 \delta G(x) dx)$ .

## 2. Polarized gluon distributions

In this talk, we discuss the  $x$  dependence of the polarized gluon distribution  $\delta G(x)$ . Let us start with the functional form of  $\delta G(x)$ . By taking account of the plausible behavior of  $\delta G(x)$  near  $x \approx 0$  and  $x \approx 1$ , we assume

$$\delta G(x) = G_+(x) - G_-(x) = B x^\gamma (1-x)^p (1+Cx), \quad (1)$$

where  $G_+(x)$  and  $G_-(x)$  are the gluon distributions with helicity parallel and antiparallel to the proton helicity, respectively. We further assume for simplicity  $G_+(x) \approx G_-(x)$  at large  $x$  and take  $C = 0$ . Then we have two unknown parameters,  $\gamma$  and  $p$ .  $B$  is determined from the normalization,  $\Delta G = 5.32$ [7], which is derived from the SMC data. We are interested in the behavior of  $\delta G(x)$  under the condition of large  $\Delta G$  ( $= 5.32$ ) and search the allowable region of  $\gamma$  and  $p$ . In order to do this, we require the positivity condition of distribution functions and utilize the recent results of several polarization experiments.

## 3. Restriction on the $x$ dependence of $\delta G(x)$

As a preliminary, to examine the behavior of  $\delta G(x)$  in eq.(1) for various values of  $\gamma$  and  $p$ ,  $\gamma$  is varied from  $-0.9$  to  $0.3$  at intervals of  $0.3$  while  $p$  is chosen independently as  $5, 10, 15, 17$  and  $20$ .

(i) First, let us consider the positivity condition of distribution functions to restrict  $\gamma$  and  $p$ . In the same way as in eq.(1) for the polarized gluon distribution, the unpolarized gluon distribution  $G(x)$  is assumed as

$$G(x) = G_+(x) + G_-(x) = \frac{A}{x^\alpha} (1-x)^k. \quad (2)$$

Since  $G_+(x)$  and  $G_-(x)$  are both positive, we obtain from eqs.(1) and (2)[7]

$$|\Delta G| \leq \frac{\Gamma(\gamma+1) \Gamma(p+1) \Gamma(k+3-\alpha) (\alpha+\gamma+p-k)^{\alpha+\gamma+p-k}}{\Gamma(\gamma+p+2) \Gamma(k+1) \Gamma(2-\alpha) (\alpha+\gamma)^{\alpha+\gamma} (p-k)^{p-k}} \int_0^1 x G(x) dx. \quad (3)$$

To restrict the region of  $\gamma$  and  $p$  from this inequality (3) with  $\Delta G = 5.32$ , we need to know the value of  $\alpha$  and  $k$  in  $G(x)$  and the integral value of  $xG(x)$  as well. As for the  $x$  dependence of  $G(x)$ , using experimental data of  $J/\psi$  productions for unpolarized muon-nucleon scatterings[8, 9], we have two possible types of parameterization of

$G(x)$  at  $Q^2 \simeq M_{J/\psi}^2$  GeV<sup>2</sup>(see Fig.1),

$$\text{Type A} \quad G(x) = 3.35 \frac{1}{x} (1-x)^{5.7}, \quad (4)$$

$$\text{Type B} \quad G(x) = 2.36 \frac{1}{x^{1.08}} (1-x)^{4.65}. \quad (5)$$

For Type A,  $\alpha$  is taken to be 1 by considering the ordinary Pomeron P, and parametrized so as to fit the data. On the other hand,  $\alpha$  is chosen to be 1.08 in Type B which is recently derived from the analysis of the experimental data of the total cross section[10]. Inserting these parameters into inequality (3) with  $\Delta G = 5.32$ , the allowed regions of  $\gamma$  and  $p$  are obtained(see Table 1 and Fig.2). In Fig.2, the region below solid or dashed lines is excluded by (3). We see that a wide region of  $\gamma$  and  $p$  which satisfies the SMC data and the positivity condition simultaneously, is allowable.

(ii) Secondly, to restrict further the allowable region of  $\gamma$  and  $p$ , we compare our model calculations with the two-spin asymmetries  $A_{LL}^{\pi^0}(\vec{p}p)$  for inclusive  $\pi^0$ -productions measured by E581/704 Collaboration using polarized (anti-)proton beams and polarized proton targets[11]. Taking  $\delta G(x)$  with the sets of  $(\gamma, p)$  allowed by the criterion of positivity, we calculate numerically  $A_{LL}^{\pi^0}(\vec{p}p)$ (see Table 2 and Fig.3). In doing so, polarized quark distributions  $\delta q(x)$  ( $q = u, d, s$ ) are necessary and taken from ref.[4]. In order to reproduce the experimental data,  $x\delta G(x)$  must have a peak at a smaller  $x$  than 0.05 and decrease very rapidly with  $x$ .

(iii) Finally, we look into the spin-dependent structure function of proton  $g_1^p(x)$ [2] and that of deuteron  $g_1^d(x)$ [12]. The results calculated by using our  $\delta q(x)$  and  $\delta G(x)$  with  $(\gamma, p)$  having survived the criteria of cases (i) and (ii) are shown in Fig.4. Furthermore, to examine the adequacy of calculations more objectively,  $\chi^2/\text{DOF}$  and the confidence level for  $xg_1^p(x)$  and  $xg_1^d(x)$  are calculated and shown in Table 3.

#### 4. Conclusion and discussion

Within the models with large  $\Delta G$  ( $= 5.32$ ), we have studied the shape of the polarized gluon distribution. By using the positivity condition of distribution functions together with the experimental data on the two-spin asymmetries  $A_{LL}^{\pi^0}(\vec{p}p)$  and the spin-dependent structure functions of  $g_1^p(x)$  and  $g_1^d(x)$ , we have restricted the  $x$  dependence of  $\delta G(x)$  given in eq.(1). As for the values of  $\gamma$  and  $p$ ,  $\gamma \lesssim -0.3$  and  $p \gtrsim 10$  seem favorable in our analysis. If  $\gamma$  and  $p$  are fixed in this region, for

example, as  $\gamma = -0.6$  and  $p = 17$ ,

$$\delta G(x) = 9.52 x^{-0.6} (1-x)^{17}, \quad (6)$$

one can reproduce all existing data quite successfully. Needless to say, with large  $\Delta G (= 5.32)$  the  $\Delta s$  becomes small and can be reconciled with the bound  $|\Delta s| \leq 0.05^{+0.02}_{-0.05}$ . In the Regge terminology,  $\gamma = -0.6$  happens to be closer to the one for unpolarized valence quark distributions rather than for unpolarized gluon distributions[13], and  $p$  seems to be inconsistent with the prediction of counting rules[14]. At present, we do not know the theoretical ground on the origin of these values of  $\gamma$  and  $p$ . Furthermore, if  $\Delta G$  is so large ( $\approx 5 \sim 6$ ), we are to have an approximate relation  $\langle L_Z \rangle_{q+G} \approx -\Delta G$  from the proton spin sum rule,  $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \langle L_Z \rangle_{q+G}$ , where  $\frac{1}{2}\Delta\Sigma$  represents the sum of the spin carried by quarks. Unfortunately, nobody knows the underlying physics of it. These are still problems to be solved even though the idea of the  $U_A(1)$  anomaly is attractive.

It is interesting to comment on another approach which has mentioned to this problem. Recently, Brodsky, Burkardt and Schmidt (BBS)[15] have proposed an interesting model of the polarized gluon distribution which incorporates color coherence and the counting rule at small and large  $x$ . At  $x \approx 0$ , the color coherence argument gives  $\delta G(x)/G(x) \approx \frac{x}{3}\langle \frac{1}{y} \rangle$  with  $\langle \frac{1}{y} \rangle \simeq 3$ , where  $\langle \frac{1}{y} \rangle$  presents the first inverse moment of the quark light-cone momentum fraction distributions in the lowest Fock state of the proton, and leads to a relation  $\gamma = -\alpha + 1$ [15, 16]. Then, contrary to our result,  $\gamma \lesssim -0.3$ , they have taken  $\gamma = 0$  by choosing  $\alpha = 1$  which is an ordinary Pomeron intercept value. Although the integrated value of  $\delta G(x)$  in the BBS model is small such as  $\Delta G = 0.45$ , the model reproduces well the EMC data  $g_1^p(x)$ ,  $g_1^n(x)$  and  $g_1^d(x)$ . In addition, we calculated  $A_{LL}^{\pi^0}(\bar{p}p)$  by using the BBS model and found that the model could reproduce  $A_{LL}^{\pi^0}(\bar{p}p)$  while the predicted value of  $A_{LL}^{\pi^0}(pp)$  slightly deviated from the data[17]. The BBS model which has small  $\Delta G (= 0.45)$  seems to be an alternative to our model with large  $\Delta G (= 5.32)$ , though in the BBS model  $\Delta s$  becomes large and cannot be reconciled with the bound  $|\Delta s| \leq 0.05^{+0.02}_{-0.05}$ .

It is very important to know the behavior of  $\delta G(x)$  and  $\delta s(x)$  independently in order to understand the spin structure of the nucleon. However the polarization experiments are still in their beginning and the form of these functions is not yet clear. We hope they will be determined in the forthcoming experiments.

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		p				
		5	10	15	17	20
$\gamma$	0.3	*	×	×	×	×
	0.0	*	×	○	○	○
	-0.3	*	○	○	○	○
	-0.6	*	○	○	○	○
	-0.9	*	○	○	○	○

		p				
		5	10	15	17	20
$\gamma$	0.3	×	×	×	×	×
	0.0	×	×	×	○	○
	-0.3	×	○	○	○	○
	-0.6	×	○	○	○	○
	-0.9	○	○	○	○	○

**Table 1:** The various sets of  $\gamma$  and  $p$  which we have examined. The circles denote the sets allowed from (3) whereas the crosses present the ones excluded from (3). The left-side (right-side) table corresponds to Type A (Type B) of  $G(x)$ . In the left-side table, the sets of  $(\gamma, p)$  with  $p = 5$ , which are filled with asterisks, are not used for calculations because  $p - k$  should be larger than zero.

		p				
		5	10	15	17	20
$\gamma$	0.3	—	—	—	—	—
	0.0	—	—	×	×	○
	-0.3	—	×	×	○	○
	-0.6	—	×	○	○	○
	-0.9	×	○	○	○	○

		p				
		5	10	15	17	20
$\gamma$	0.3	—	—	—	—	—
	0.0	—	—	○	○	○
	-0.3	—	×	○	○	○
	-0.6	—	○	○	○	○
	-0.9	×	○	○	○	○

**Table 2:** The various sets of  $\gamma$  and  $p$  which we have examined. The circles denote the sets allowed from the  $A_{LL}^{\pi^0}$  whereas the crosses present the ones excluded from it. The minuses denote the sets excluded from Table 1. The left-side (right-side) table corresponds to the  $A_{LL}^{\pi^0}$  for  $pp$  collisions ( $\bar{p}p$  collisions).

(A)

		p			
		10	15	17	20
$\gamma$	0.0	—	—	—	0.766 (83.5%)
	−0.3	—	—	0.501 (99.3%)	0.444 (99.8%)
	−0.6	—	0.444 (99.8%)	0.448 (99.8%)	0.474 (99.6%)
	−0.9	0.646 (94.5%)	0.728 (87.7%)	0.771 (82.9%)	0.839 (73.3%)

(B)

		p			
		10	15	17	20
$\gamma$	0.0	—	—	—	1.006 (43.8%)
	−0.3	—	—	0.726 (71.5%)	0.629 (80.6%)
	−0.6	—	0.539 (87.9%)	0.501 (90.4%)	0.463 (92.7%)
	−0.9	0.581 (84.7%)	0.548 (87.2%)	0.547 (87.2%)	0.554 (86.7%)

**Table 3:** The values of  $\chi^2/\text{DOF}$  and the confidence level (in the parentheses) for  $(\gamma, p)$  which have survived in Table 2. (A) ((B)) corresponds to the  $xg_1^p(x)$  ( $xg_1^d(x)$ ). The DOF of  $\chi^2$  to  $xg_1^p(x)$  ( $xg_1^d(x)$ ) is 34 (11). The minus signs denote the sets excluded from Tables 1 and 2.



## Figure captions

**Fig. 1:** The parametrization of the gluon distribution functions  $xG(x, Q^2)$  at  $Q^2 \approx 10\text{GeV}^2$ . The solid (dashed) line denotes Type A (B). The data of opened (closed) circles are taken from [8] ([9]).

**Fig. 2:** The allowed region by (3) for  $\gamma$  and  $p$ . The solid (dashed) line corresponds to Type A (B). The region below the lines are excluded.

**Fig. 3:** The produced  $\pi^0$  transverse momenta  $p_T$  dependence of  $A_{LL}^{\pi^0}(\vec{p}p)$  for various  $p$  ( $= 5 - 20$ ) with (a)  $\gamma = 0$  and (b)  $\gamma = -0.9$ . Data are taken from [11].

**Fig. 4:** The  $x$  dependence of  $xg_1^p(x)$  and  $xg_1^d(x)$  for various  $p$  ( $= 5 - 20$ ) with (a)  $\gamma = 0$  and (b)  $\gamma = -0.9$ . The data of  $xg_1^p(x)$  ( $xg_1^d(x)$ ) are taken from [2, 1] ([12]).

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